Transverse Deformation of Parton Distributions and Transversity Decomposition of Angular Momentum

Matthias Burkardt
Department of Physics, New Mexico State University, Las Cruces, NM 88003-0001, U.S.A.
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Impact parameter dependent parton distributions are transversely distorted when one considers transversely polarized nucleons and/or quarks. This provides a physical mechanism for the T-odd Sivers effect in semi-inclusive deep-inelastic scattering. The transverse distortion can also be connected with Ji's quark angular momentum relation. The distortion of chirally odd impact parameter dependent parton distributions is related to chirally odd GPDs. This result is used to provide a decomposition of the quark angular momentum w.r.t. quarks of definite transversity. Chirally odd GPDs can thus be used to determine the correlation between quark spin and quark angular momentum in unpolarized nucleons. Based on the transverse distortion, we also suggest a qualitative connection between chirally odd GPDs and the Boer-Mulders effect.

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I. INTRODUCTION

During the last few years, important breakthroughs have been made in our understanding of T-odd single-spin asymmetries (SSA) in semi-inclusive deep-inelastic scattering (SIDIS) [1, 2]. In a seminal, paper Brodsky, Hwang and Schmidt [3], provided a simple model calculation in which the interference of final state interaction (FSI) phases between different partial waves gave rise to a nontrivial Sivers effect [4]. This calculation clearly demonstrated that T-odd distributions can also survive in the Bjorken limit in QCD. Following this work, the connection between these FSI phases and the Wilson line gauge links in gauge invariantly defined unintegrated parton densities was recognized [5, 6]. This also led to the prediction that, up to a sign, the Sivers functions in SIDIS and polarized Drell-Yan are the same [5]. Soon later, an intuitive connection between the sign of the Sivers effect and the transverse distortion of impact parameter dependent parton distributions in transversely polarized targets was proposed [7]. This connection also explained the similarity between the light-cone overlap integrals relevant for the Sivers effect and for the anomalous magnetic moment [8].

Generalized parton distributions (GPDs) provide a decomposition of form factors at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\begin{aligned} \int dx H_q(x,\xi,t) &= F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t) \\ \int dx E_q(x,\xi,t) &= F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t), \end{aligned} \tag{1}$$

where x_i and x_f are the momentum fractions of the quark before and after the momentum transfer. $2\xi = x_f - x_i$ represents their difference. For recent reviews, with more precise definitions and a detailed discussion of their early history, the reader is referred to Refs. [9, 10, 11, 12]. $F_1^q(t)$, $F_2^q(t)$, $G_A^q(t)$, and $G_P^q(t)$ are the Dirac, Pauli, axial, and pseudoscalar formfactors, respectively. Note that the measurement of the quark momentum fraction x singles out one space direction (the direction of the momentum). Therefore, it makes a difference whether the momentum transfer is parallel, or perpendicular to this momentum. The GPDs must therefore depend on an additional variable which characterizes the direction of the momentum transfer relative to the momentum of the active quark. Usually, one parameterizes this dependence through the dimensionless variable ξ. Throughout this work we will focus on the limiting case $\xi = 0$, where GPDs can be interpreted as the Fourier transform of the distribution of partons in the transverse plane (see Refs. [13, 14, 15, 16, 17] and references therein).

The impact parameter dependent distributions are defined as follows. First one introduces nucleon states which are localized in transverse position space at $\mathbf{R}_\perp$ (they are eigenstates of the transverse center of momentum with eigenvalue $\mathbf{R}_\perp$)

$$\left|p^+, \mathbf{R}_\perp, \lambda\right\rangle = \mathcal{N} \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} e^{-i\mathbf{p}_\perp \mathbf{R}_\perp} \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle \tag{2}$$

where $\mathcal{N}$ is some normalization factor. In these localized states, the impact parameter dependent distributions are then defined as the familiar light-cone correlations. For further details, see Refs. [14, 16].

For example, for the impact parameter dependent distribution $q(x, \mathbf{b}_\perp)$ of unpolarized quarks in an unpolarized target one finds for the distribution of quarks with momentum fraction x

$$q(x, \mathbf{b}_\perp) = \mathcal{H}(x, \mathbf{b}_\perp) \equiv \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \mathbf{\Delta}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2), \tag{3}$$

where $\mathbf{b}_\perp$ is the transverse distance from the active quark to the transverse center of momentum

$$\mathbf{R}_\perp = \sum_i x_i \mathbf{r}_{\perp,i}. \tag{4}$$

and $\mathbf{\Delta}_\perp = \mathbf{p}'_\perp - \mathbf{p}_\perp$. The transverse center of momentum is the analog of the nonrelativistic center of mass and the sum in Eq. (4) extends over both quarks and gluons. The x_i are the momentum fractions of each parton which play the same role that the mass fraction plays in nonrelativistic physics. One of the remarkable features of Eq. (3) is that there are no relativistic corrections to the interpretation of GPDs (at $\xi = 0$) as Fourier transforms of parton distributions in impact parameter space [7]. This is due to the presence of a Galilean subgroup of transverse boosts in the light front formulation of relativistic dynamics [18]. Another remarkable feature is that the impact parameter dependent parton distributions obtained via Eq. (3) have a probabilistic interpretation and satisfy corresponding positivity constraints [19].

For the distribution $q_X(x, \mathbf{b}_\perp)$ of unpolarized quarks in a nucleon state that is a superposition of positive and negative (light-cone) helicity states

$$|X\rangle \equiv \frac{1}{\sqrt{2}} \left[\left| p^+, \mathbf{R}_\perp, + \right\rangle + \left| p^+, \mathbf{R}_\perp, - \right\rangle \right] \tag{5}$$

one finds (for details see Ref. [14], where a detailed definition of these distributions is provided)

$$q_X(x, \mathbf{b}_\perp) = \mathcal{H}(x, \mathbf{b}_\perp) - \frac{1}{2m} \frac{\partial}{\partial b_y} \mathcal{E}(x, \mathbf{b}_\perp) \tag{6}$$

where

$$\mathcal{E}(x, \mathbf{b}_\perp) \equiv \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \mathbf{\Delta}_\perp} E^q(x, 0, -\mathbf{\Delta}_\perp^2). \tag{7}$$

Up to relativistic corrections, due to the transverse localization of the wave packet (these corrections will be discussed below in connection with the angular momentum relation), this state can be interpreted as a transversely polarized target and Eq. (7) predicts that the impact parameter dependent parton distributions give rise to a transverse flavor dipole moment in a transverse polarized target. The average magnitude of this distortion is normalized to the anomalous magnetic moment contribution from that quark flavor [14]. The physical origin of this distortion is the fact that the virtual photon in DIS couples only to the $j^+ = j^0 + j^3$ component of the quark density in the Bjorken limit. For quarks with nonvanishing orbital angular momentum, the j^3 component of the quark current has a left-right asymmetry due to the orbital motion [20].

The transverse distortion of the parton distributions exhibited in Eq. (6), in combination with an attractive final state interaction, has been suggested as a simple explanation for the Sivers effect in QCD [21, 22]. Since the sign of E^q can be related to the contribution from quark flavor q to the anomalous magnetic moment of nucleons, Eq. (6) has been the basis for a prediction of the signs of the Sivers effect for u and d quarks [21, 22], which have been confirmed by the HERMES collaboration [23].

In this work we will first discuss the connection between the transverse distortion of GPDs and Ji's quark angular momentum relation [24]. This will allow us to draw a link between chirally odd GPDs and the correlation between the angular momentum and spin of the quarks. We also propose a simple explanation for the Boer-Mulders effect [25], where the asymmetry arises from the transverse distortion of chirally odd GPDs.

II. TRANSVERSE COMPONENT OF THE ANGULAR MOMENTUM

In this section, we discuss the connection between the transverse distortion of quark distribution in impact parameter space and Ji's quark angular momentum relation

$$J^i = \frac{1}{2} \varepsilon^{ijk} \int d^3x M^{0jk}. \tag{8}$$

The angular momentum density $M^{\alpha\mu\nu} = T^{\alpha\nu}x^\mu - T^{\alpha\mu}x^\nu$ is expressed in terms of the energy momentum tensor $T^{\mu\nu}$.

Since the angular momentum operator is expressed in terms of the position space moments of the energy momentum tensor, it is possible to relate J_q to the form factor of the energy momentum tensor [24]

$$\left\langle p' \left| T_q^{\mu\nu} \right| p \right\rangle = \bar{u}(p') \left[A_q(\Delta^2)\gamma^\mu \bar{p}^\nu + B_q(\Delta^2)\frac{i\sigma^{\nu\alpha}}{2M}\bar{p}^\mu \Delta_\alpha + C_q(\Delta^2)\frac{(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)}{M} + \bar{C}_q(\Delta^2) g^{\mu\nu} M \right] u(p), \tag{9}$$

where symmetrization of the indices μ and ν is implicit and $2\bar{p}^\mu = p^\mu + p'^\mu$. The label q distinguishes the form factors of the different quark flavors or the glue. The angular momentum relation obtained from Eqs. (8) and (9) reads [24]

$$\left\langle J_q^i \right\rangle = S^i \left[A_q(0) + B_q(0) \right], \tag{10}$$

where S^i is the nucleon spin. Further details can be found in Ref. [24].

For the relation between the transverse deformation of impact parameter dependent parton distributions and the angular momentum, we now concentrate on the form factor of the 'good' component of the energy momentum tensor

$$\langle p' | T_q^{++}(0) | p \rangle = \bar{u}(p') \left[A_q(-\mathbf{\Delta}_\perp^2)\gamma^+ p^+ + B_q(-\mathbf{\Delta}_\perp^2) p^+ \frac{i\sigma^{+i}\Delta_i}{2M} \right] u(p). \tag{11}$$

In this work, we are mainly interested in the spin component perpendicular to the light-cone direction, which is sensitive to longitudinal boosts. It is thus important to specify the frame. Here and in the following we only consider the special case $p^+ = p'^+ = M$, i.e. our results apply to the rest frame of the target.

Application of Eq. (11) to a delocalized wave packet $|\psi\rangle$ of a transversely polarized nucleon with transverse spin S^j yields

$$\left\langle \psi \left| \int d^2\mathbf{b}_\perp b_i T_q^{++}(\mathbf{b}_\perp) \right| \psi \right\rangle = \mathcal{N} \varepsilon_{ij} S^j p^+ \left[A_q(0) + B_q(0) \right], \tag{12}$$

where $\mathcal{N}$ is a normalization factor depending on the wave packet. Ideally, we would like to take the expectation value in Eq. (12) in plane wave state with $\vec{p} = 0$, but this leads to ill defined expressions when $b \to \infty$. In order to regularize these expressions, we thus use $|\psi\rangle = \int d^3k\, \psi(\vec{k}) \left| \vec{k}, \vec{S} \right\rangle$, where $\left| \vec{k}, \vec{S} \right\rangle$ are spin eigenstates and imagine taking the limit where $\psi(\vec{k})$ is nonzero only for $\vec{k} = 0$ in the end of the calculation. In the light-cone analysis, we have in mind taking the limit $k^z = 0$ immediately and choosing a dependence on $\mathbf{k}_\perp$ that is axially symmetric (depends only on $|\mathbf{k}_\perp|^2$). For the discussion in a general frame, $\psi(\vec{k}) = \psi(\vec{k}^2)$. the actual shape of the wave functions entering these wave packets is irrelevant after the limit $\vec{k} \to 0$ has been taken. A comparison betwwen Eq. (12) and Eq. (10) shows that the expectation value of the angular momentum of the quarks $\left\langle J_q^i \right\rangle$ in a transversely polarized delocalized state can be related to the transverse center of momentum of the quarks in the same state. This observation provides a physical explanation for Ji's result linking J_q and the GPDs $H_q(x,0,0)$ and $E_q(x,0,0)$.

Moreover, in light-cone gauge $A^+ = 0$, T^{++} contains no interactions between the fields and it is natural to decompose

$$T^{++} = T_q^{++} + T_g^{++} = i q_+^\dagger \partial^+ q_+ + Tr\left(\partial^+ \vec{\mathbf{A}}_\perp^2 \right), \tag{13}$$

where $q_+ = \frac{1}{2}\gamma^-\gamma^+ q$ is the "good" component. This provides a parton model interpretation for Ji's quark angular momentum relation. Upon switching to a mixed representation (momentum/ position space representation for the longitudinal/transverse coordinate respectively) and express the transverse shift of the center of momentum for a particular quark flavor in terms of the impact parameter dependent parton distributions, yielding

$$\left\langle \psi \left| J_q^i \right| \psi \right\rangle = \varepsilon^{ij} M \int dx \int d^2\mathbf{b}_\perp q_\psi(x, \mathbf{b}_\perp) x b^j, \tag{14}$$

where $q_\psi(x, \mathbf{b}_\perp)$ is the impact parameter dependent parton distribution evaluated in the state ψ.

We should emphasize that it is crucial for this argument that we work with a delocalized state which is centered around the origin. As a counter example, when As an application of Eq. (14), we now insert $q_X(x, \mathbf{b}_\perp)$ for the "transversely polarized" state above (6), yielding

$$\left\langle X \left| J_q^x \right| X \right\rangle = \frac{1}{2} \int dx E_q(x,0,0) x, \tag{15}$$

which is obviously only part of Eq. (10). In order to better understand the connection between impact parameter dependent PDFs and the angular momentum of the quarks, we now investigate the origin of this discrepancy further. For this purpose we note that the state $|X\rangle$ (5) is localized in impact parameter space, and its momentum space wave packet contains an integral over transverse momentum. However, for states with a nonzero transverse momentum, the light-front helicity eigenstates and the rest frame spin eigenstates are not the same. Indeed, already for $|\mathbf{k}_\perp| \ll M$ one finds [26]

$$|k,+\rangle_I = |k,+\rangle_F - \frac{k_R}{2M}|k,-\rangle_F \tag{16}$$

$$|k,-\rangle_I = |k,-\rangle_F + \frac{k_L}{2M}|k,+\rangle_F \tag{17}$$

where $k_R = k_1 + ik_2$ and $k_L = k_1 - ik_2$. The subscripts "I" and "F" refer to the spin eigenstates in the instant as well as front form of dynamics respectively [18]. Explicit representations for instant and front form spinors and the transformation relating them can also be found in the appendix of Ref. [10]. As a consequence of this "Melosh rotation", one should only identify the state $|X\rangle$ with a state that is transversely polarized in its rest frame up to relativistic corrections.

This well-known result has important consequences if we consider a delocalized state that is polarized in the $+\hat{x}$ direction in the rest frame

$$\left|\psi_I^{+\hat{x}}\right\rangle \equiv \int d^2\mathbf{k}_\perp \psi_I^{+\hat{x}}(\mathbf{k}_\perp)\,|\mathbf{k}_\perp,+\hat{x}\rangle_I \tag{18}$$

$$= \int d^2\mathbf{k}_\perp \left[\psi_F^{+\hat{x}}(\mathbf{k}_\perp)\,|\mathbf{k}_\perp,+\hat{x}\rangle_F + \psi_F^{-\hat{x}}(\mathbf{k}_\perp)\,|\mathbf{k}_\perp,-\hat{x}\rangle_F\right] \tag{19}$$

with

$$\psi_F^{+\hat{x}}(\mathbf{k}_\perp) = \left(1 - i\frac{k_2}{2m}\right)\psi_I^{+\hat{x}}(\mathbf{k}_\perp) \tag{20}$$

$$\psi_F^{-\hat{x}}(\mathbf{k}_\perp) = \frac{k_1}{2M}\psi_I^{+\hat{x}}(\mathbf{k}_\perp).$$

For the wave packet $\psi_I^{+\hat{x}}(\mathbf{k}_\perp)$ in the rest frame we have in mind an axially symmetric function that describes a state that is delocalized in transverse position space, but centered around the origin. The longitudinal momentum $k^+ = k^0 + k^3 \approx M$ is kept fixed.

It is fallacious to believe that this effect is negligible in the limiting case of a delocalized wave packet. Indeed, to leading order in $\frac{1}{M}$ (higher orders in $\frac{1}{M}$ involve additional powers of the size R of the wave packet in the denominator and are suppressed for a large wave packet), the factor $\left(1 - i\frac{k_2}{2M}\right)$ implies that the corresponding position space wave packet in the front form is shifted sideways by half a Compton wavelength

$$\tilde{\psi}_F^{+\hat{x}}(\mathbf{b}_\perp) = \tilde{\psi}_I^{+\hat{x}}\left(\mathbf{b}_\perp - \frac{1}{2M}\hat{\mathbf{y}}\right). \tag{21}$$

To leading order in $\frac{1}{M}$ there is no significant effect from $\psi_F^{-\hat{x}}$, since all contributions to the center of momentum are proportional to $\left|\tilde{\psi}_F^{-\hat{x}}(\mathbf{b}_\perp)\right|^2 \sim \frac{1}{M^2}$, i.e. for dimensional reasons they must also be or order $\frac{1}{R}$. The sideways shift implies that a large axially symmetric wave packet for a spin $\frac{1}{2}$ particle, polarized in the $+\hat{x}$ direction, that is centered around the origin in the rest frame corresponds again to a particle polarized in the $+\hat{x}$ direction in the front form, but now the wave packet is centered around $\mathbf{b}_\perp = +\frac{1}{2M}\hat{\mathbf{y}}$. For a particle that is polarized in the $-\hat{x}$ direction the shift is in the opposite direction.

This phenomenon has a number of applications. First it explains how an elementary Dirac particle, for which

$$q_X(x,\mathbf{b}_\perp) = \delta(x-1)\delta(\mathbf{b}_\perp) \tag{22}$$

can yield a nontrivial result for its total angular momentum from Eq. (14): For a state that is polarized in the $+\hat{x}$ direction, and which in the instant form is described by a wave packet $\tilde{\psi}_{+\hat{x}}(\mathbf{b}_\perp)$, the corresponding front form wave packet is centered around $\mathbf{b}_\perp = +\frac{1}{2M}\hat{y}$. In general, the distribution of partons in a wave packet $q_\psi(x,\mathbf{b}_\perp)$ is obtained by convoluting the intrinsic distribution $q(x,\mathbf{b}_\perp)$ (relative to the center of momentum) with the distribution $|\psi(\mathbf{b}_\perp)|^2$ resulting from the wave packet. For our example of an elementary Dirac particle, this implies

$$q_\psi(x,\mathbf{b}_\perp) = \int d^2\mathbf{r}_\perp \left|\tilde{\psi}_F^{+\hat{x}}(\mathbf{r}_\perp)\right|^2 q(x,\mathbf{b}_\perp+\mathbf{r}_\perp) = \delta(x-1)\left|\tilde{\psi}_F^{+\hat{x}}(\mathbf{b}_\perp)\right|^2 = \delta(x-1)\left|\tilde{\psi}_I^{+\hat{x}}(\mathbf{b}_\perp - \frac{1}{2M}\hat{\mathbf{y}})\right|^2 \tag{23}$$

plus corrections that are negligible for a large wave packet. upon integrating over $\mathbf{b}_\perp$, one easily finds

$$M\int d^2\mathbf{b}_\perp q_\psi(x,\mathbf{b}_\perp)\mathbf{b}_\perp = \frac{1}{2}\delta(x-1) \tag{24}$$

and therefore

$$\left\langle J_q^x\right\rangle = M\int dx\int d^2\mathbf{b}_\perp q_\psi(x,\mathbf{b}_\perp)b^y = \frac{1}{2}. \tag{25}$$

From the derivation it should be clear that the sideways shift by $b^y=\frac{1}{2M}$ is essential for this result.

The second application is to a spin $\frac{1}{2}$ particle polarized in the $+\hat{x}$ direction with a nontrivial intrinsic distribution $q_X(x,\mathbf{b}_\perp)=\mathcal{H}(x,\mathbf{b}_\perp)-\frac{1}{2M}\frac{\partial}{\partial b_y}\mathcal{E}(x,\mathbf{b}_\perp)$. In this case

$$q_\psi(x,\mathbf{b}_\perp) = \int d^2\mathbf{r}_\perp\left|\tilde{\psi}_F^{+\hat{x}}(\mathbf{r}_\perp)\right|^2 q(x,\mathbf{b}_\perp+\mathbf{r}_\perp) \tag{26}$$

and the resulting transverse flavor dipole moment receives contributions both from the wave packet as well as from the intrinsic distortion. After an appropriate shift of variables one easily finds

$$\begin{aligned}\int d^2\mathbf{b}_\perp q_\psi(x,\mathbf{b}_\perp)\mathbf{b}_\perp &= \int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp)\int d^2\mathbf{r}_\perp\left|\tilde{\psi}_F^{+\hat{x}}(\mathbf{r}_\perp)\right|^2\mathbf{r}_\perp+\int d^2\mathbf{r}_\perp\left|\tilde{\psi}_F^{+\hat{x}}(\mathbf{r}_\perp)\right|^2\int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp)\mathbf{b}_\perp \\ &= \frac{1}{2M}\left[H(x,0,0)+E(x,0,0)\right].\end{aligned} \tag{27}$$

From the point of view of impact parameter dependent PDFs, the $H(x,0,0)$ contribution in Ji's relation is thus due to the sideways shift of the wave packet in the transition from the instant form to the front form description

$$\left\langle J_q^x\right\rangle = M\int dx\int d^2\mathbf{b}_\perp q_\psi(x,\mathbf{b}_\perp)\mathbf{b}_\perp = \frac{1}{2}\int dx\left[H(x,0,0)+E(x,0,0)\right]x. \tag{28}$$

Although, due to rotational invariance, the final result holds for any component of $\vec{J}_q$, in the context of impact parameter dependent parton distributions the Ji relation naturally emerges as a relation for $J_q^\perp$. In particular it appears natural to identify the integrand of Eq. (28) $\frac{1}{2}\left[H(x,0,0)+E(x,0,0)\right]x$ with a momentum decomposition of the transverse component of the quark angular momentum in a transverse polarized target. The term containing $E(x,0,0)$ arises from the transverse deformation of GPDs in the center of momentum frame, while the term containing $H(x,0,0)$ in Ji's relation arises from an overall transverse shift when going from transverse polarized nucleons in the instant form (rest frame) to the front form (infinite momentum frame).

III. CHIRALLY ODD GPDS

Similar to the chirally even case, chirally odd GPDs are defined as non-forward matrix elements of light-like correlation functions of the tensor charge

$$\begin{aligned}p^+\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\left\langle p'\right|\bar{q}\left(-\frac{z}{2}\right)\sigma^{+j}\gamma_5 q\left(\frac{z}{2}\right)\left|p\right\rangle &= H_T(x,\xi,t)\bar{u}\sigma^{+j}\gamma_5 u+\tilde{H}_T(x,\xi,t)\varepsilon^{+j\alpha\beta}\bar{u}\frac{\Delta_\alpha p_\beta}{M^2}u \\ &\quad +E_T(x,\xi,t)\varepsilon^{+j\alpha\beta}\bar{u}\frac{\Delta_\alpha\gamma_\beta}{2M}u+\tilde{E}_T(x,\xi,t)\varepsilon^{+j\alpha\beta}\bar{u}\frac{p_\alpha\gamma_\beta}{M}u.\end{aligned} \tag{29}$$

The connection between GPDs at $\xi=0$ and parton distributions in impact parameter space has recently been extended to the chirally odd sector [27]. Quarks $q(x,\mathbf{b}_\perp,\mathbf{s})$ with transverse polarization $\mathbf{s}=(\cos\chi,\sin\chi)$ are projected out by the operator $\frac{1}{2}\bar{q}\left[\gamma^+-s^j i\sigma^{+j}\gamma_5\right]q$. Even for an unpolarized target, the transversity density $\delta^i q(x,\mathbf{b}_\perp)$, obtained from

$$s_i\delta^i q(x,\mathbf{b}_\perp) = q(x,\mathbf{b}_\perp,\mathbf{s})-q(x,\mathbf{b}_\perp,-\mathbf{s}), \tag{30}$$

can be nonzero. Indeed, in Ref. [27] it is shown that

$$\delta^i q(x,\mathbf{b}_\perp)\equiv-\frac{\varepsilon^{ij}}{2M}\frac{\partial}{\partial b_j}\left[2\tilde{\mathcal{H}}_T(x,\mathbf{b}_\perp)+\mathcal{E}_T(x,\mathbf{b}_\perp)\right], \tag{31}$$

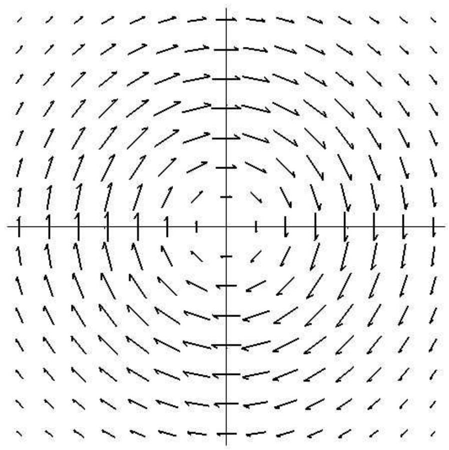

FIG. 1: Distribution of transversity in impact parameter space for a simple model (34).

and

$$\tilde{\mathcal{H}}_T(x,\mathbf{b}_\perp) \equiv \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp} \tilde{H}_T(x,0,-\mathbf{\Delta}_\perp^2) \tag{32}$$

$$\mathcal{E}_T(x,\mathbf{b}_\perp) \equiv \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp} E_T(x,0,-\mathbf{\Delta}_\perp^2)$$

Eq. (31) exhibits a nontrivial flavor dipole moment perpendicular to the quark spin

$$\int d^2\mathbf{b}_\perp \delta^i q(x,\mathbf{b}_\perp) b_j = \frac{\varepsilon^{ij}}{2M}\left[2\tilde{H}_T(x,0,0)+E_T(x,0,0)\right]. \tag{33}$$

The resulting effect is best illustrated in a simple model [Fig. 1]

$$2\tilde{\mathcal{H}}_T(x,\mathbf{b}_\perp)+\mathcal{E}_T(x,\mathbf{b}_\perp) \propto \exp(-\mathbf{b}_\perp^2). \tag{34}$$

Physically, the nonvanishing transversity density in an unpolarized target is due to spin-orbit correlations in the quark wave functions: if the quarks have orbital angular momentum then their γ^+-density is enhanced on one side, i.e. their distribution appears shifted sideways [14, 20]. For unpolarized nucleons all orientations of the orbital angular momentum are equally likely and therefore the unpolarized quark distribution is axially symmetric. However, if there is a correlation between the orientation of the quark spin and the angular momentum then quarks of a certain orientation will be shifted towards one side, while those with a different orientation will be shifted towards a different side.

IV. TRANSVERSITY DECOMPOSITION OF THE ANGULAR MOMENTUM

In the discussion about the physical origin of the transversity distribution in an unpolarized target, we hinted already at a connection between the linear combination $2\tilde{H}_T + E_T$ of chirally odd GPDs on the one hand and the correlation of quark spin and angular momentum on the other hand. In order to quantify this phenomenon, we are now considering a decomposition of the quark angular momentum with respect to quark transversity. Such a decomposition is possible since T_q^{++}, whose form factors enter Ji's quark angular momentum relation [24] does not mix quark transversity states. Indeed, if we denote positive and negative helicity states with $q_\rightarrow$ and $q_\leftarrow$ respectively, one finds

$$\begin{aligned} T_q^{++} = i\bar{q}\gamma^+ \overleftrightarrow{D}^+ q &= i\bar{q}_\rightarrow\gamma^+ \overleftrightarrow{D}^+ q_\rightarrow + i\bar{q}_\leftarrow\gamma^+ \overleftrightarrow{D}^+ q_\leftarrow \\ &= \frac{i}{2}\left(\bar{q}_\rightarrow+\bar{q}_\leftarrow\right)\gamma^+ \overleftrightarrow{D}^+ \left(q_\rightarrow+q_\leftarrow\right) + \frac{1}{2}\left(\bar{q}_\rightarrow-\bar{q}_\leftarrow\right)\gamma^+ \overleftrightarrow{D}^+ \left(q_\rightarrow-q_\leftarrow\right) = T_{q,+\hat{x}}^{++} + T_{q,-\hat{x}}^{++}. \end{aligned} \tag{35}$$

For an arbitrary transverse spin direction the decomposition reads

$$T_q^{++} = i\bar{q}\gamma^+ \overleftrightarrow{D}^+ q = \sum_{\mathbf{s}} \frac{i}{2}\bar{q}\left[\gamma^+ - s^j i\sigma^{+j}\gamma_5\right] \overleftrightarrow{D}^+ q = \frac{1}{2}\sum_{\mathbf{s}}\left[T_q^{++} + s^j\delta^j T_q^{++}\right] = \sum_{\mathbf{s}} T_{q,\mathbf{s}}^{++}, \tag{36}$$

where the summation is over $\mathbf{s} = \pm(\cos\chi, \sin\chi)$, and

$$\delta^j T_q^{++} \equiv \bar{q}\sigma^{+j}\gamma_5 \overleftrightarrow{D}^+ q \tag{37}$$

represents the transversity asymmetry of the momentum density. We will first present a heuristic argument for the transversity decomposition of the angular momentum based on the shift of parton distributions in impact parameter space. However, for the sake of completeness, the heuristic derivation is followed by a more formal derivation that parallels the approach chosen in Ji's original paper [24].

In the previous section we discussed that the total angular momentum carried by quarks of flavor q can be associated with the transverse shift of the center of momentum of those quarks in a target state that is described by a delocalized wave packet with transverse polarization in the rest frame. Since T_q^{++} does not mix quark transversity, we can thus use this result to provide a decomposition of the quark angular momentum into transversity eigenstates.

For an unpolarized target, one might naively suspect that there is no effect from the overall sideways shift of the transverse center of momentum discussed in Section II. However, when one considers the transversity asymmetry of the angular momentum, the contributions from the two polarizations add up. As a result, the transversity asymmetry in a delocalized target at rest contains a term $\frac{1}{4}\int dx H_T(x,0,0)\, x$. The parton model interpretation of this term is the same as the term involving $H(x,0,0)$ in Ji's relation and it results from an overall transverse displacement of the center of light-cone momentum in a state that is discribed by a delocalized wavepacket centered around the origin in the rest frame.

In addition, there is the shift of the transverse center of momentum arising from the deformation in the center of momentum frame (31). Upon inserting Eq.(31) into (14), and adding the effect from the overall shift, we find for the angular momentum carried by quarks with transverse spin in the $\mathbf{s}$-direction in an unpolarized target

$$\left\langle J_q^i(\mathbf{s})\right\rangle = \frac{1}{2}\left\langle J_q^i + s^j\delta^j J_q^i\right\rangle = \frac{s^j}{2}\left\langle \delta^j J_q^i\right\rangle = \frac{s^i}{4}\int dx\left[H_T(x,0,0) + 2\tilde{H}_T(x,0,0) + E_T(x,0,0)\right]x. \tag{38}$$

The same GPDs that describe the distribution of transversity in impact parameter space also characterize the correlation between quark spin and angular momentum in an unpolarized target.

So far we have put special emphasis on drawing a connection between the transverse distortion of impact parameter dependent PDFs and the angular momentum of the quarks. In the following we present an alternative derivation of Eq. (38) which follows more the approach in Ref. [24]. For this purpose we consider the form factor of the transversity density with one derivative [27, 28]

$$\begin{aligned}\langle p'|\,\bar{q}\sigma^{\lambda\mu}\gamma_5 i\overleftrightarrow{D}^\nu q\,|p\rangle \;=\; &\bar{u}\sigma^{\lambda\mu}\gamma_5 u\,\bar{p}^\nu A_{T20}(t) + \frac{\varepsilon^{\lambda\mu\alpha\beta}\Delta_\alpha\bar{p}_\beta\bar{p}^\nu}{M^2}\bar{u}u\,\tilde{A}_{T20}(t) + \frac{\varepsilon^{\lambda\mu\alpha\beta}\Delta_\alpha\bar{p}^\nu}{2M}\bar{u}\gamma_\beta u\,B_{T20}(t)\\ &+\frac{\varepsilon^{\lambda\mu\alpha\beta}\bar{p}_\alpha\Delta^\nu}{M}\bar{u}\gamma_\beta u\,\tilde{B}_{T21}(t)\end{aligned} \tag{39}$$

where antisymmetrization in λ and μ and symmetrization in μ and ν is implied. The invariant form factors in Eq. (39) are the second moments of the chirally odd GPDs

$$\begin{aligned} A_{T20}(t) &= \int_{-1}^{1} dx x H_T(x,\xi,t) \\ \tilde{A}_{T20}(t) &= \int_{-1}^{1} dx x \tilde{H}_T(x,\xi,t) \\ B_{T20}(t) &= \int_{-1}^{1} dx x E_T(x,\xi,t) \\ -2\xi\tilde{B}_{T21}(t) &= \int_{-1}^{1} dx x \tilde{E}_T(x,\xi,t). \end{aligned} \tag{40}$$

(41)

The projection operator on transverse spin (transversity) eigenstates $P_{\pm\hat{x}} \equiv \frac{1}{2}\left(1 \pm \gamma^x\gamma_5\right)$ commutes with both γ^0, γ^y, and γ^z. Hence neither T_q^{0y} nor T_q^{0z} mix between transversity (in the $\hat{x}$ direction) and it is possible to decompose

$$T_q^{0y} = T_{q,+\hat{x}}^{0y} + T_{q,-\hat{x}}^{0y} \tag{42}$$

w.r.t. transversity, where

$$T_{q,\pm\hat{x}}^{0y} = \frac{i}{2}\bar{q}\left[\gamma^0 D^y + \gamma^y D^0\right] P_{\pm\hat{x}} q = \frac{1}{2}\left(T_q^{0y} \pm \delta^x T_q^{0y}\right) \tag{43}$$

where

$$\delta^x T_q^{0y} = \frac{i}{2}\bar{q}\left(\gamma^0 \overleftrightarrow{D}^y + \gamma^y \overleftrightarrow{D}^0\right)\gamma^x\gamma_5 q = -\frac{1}{2}\bar{q}\left(\sigma^{x0} \overleftrightarrow{D}^y + \sigma^{xy} \overleftrightarrow{D}^0\right) q. \tag{44}$$

The same kind of decomposition can be made for T_q^{0z}. Evidently, these observations allow a similar decomposition for

$$J_q^x = \int d^3x \left(yT^{0z} - zT^{0y}\right) = J_{q,+\hat{x}}^x + J_{q,-\hat{x}}^x. \tag{45}$$

The dependence of J_q^x on the transversity of the quarks is given by

$$J_{q,\pm\hat{x}}^x = \frac{1}{2}\left(J_q^x \pm \delta^x J_q^x\right) \tag{46}$$

where

$$\delta^x J_q^x = \int d^3x \left(\delta^x T^{0z} y - \delta^x T^{0y} z\right) = \frac{1}{2}\int d^3x\, \bar{q}\left[-\left(\sigma^{x0} \overleftrightarrow{D}^z + \sigma^{xz} \overleftrightarrow{D}^0\right) y + \left(\sigma^{x0} \overleftrightarrow{D}^y + \sigma^{xy} \overleftrightarrow{D}^0\right) z\right] q. \tag{47}$$

The operators appearing in Eq. (47) correspond to the operator appearing on the l.h.s. of Eq. (39) with $\lambda = x$, $\mu = 0$, $\nu = z$, and $\lambda = x$, $\mu = 0$, $\nu = y$ respectively. For the expectation value of the transversity asymmetry $\delta^x J_q^x = J_{q,+\hat{x}}^x - J_{q,-\hat{x}}^x$, Eq. (39) thus implies in an unpolarized target at rest

$$\left\langle \delta^x J_q^x \right\rangle = \frac{1}{2}\left[A_{T20} + 2\tilde{A}_{T20}(0) + B_{T20}(0)\right], \tag{48}$$

which is the analogue of Ji's result $\left\langle J_q^i \right\rangle = S^i\left[A_{20}(0) + B_{20}(0)\right]$. The angular momentum J^x carried by quarks with transverse polarization (transversity) in the $+\hat{x}$ direction in an unpolarized target is one half of Eq. (48). Together with (40) Eq. (48) provides an independent confirmation of the main result of this paper (38). While the alternate derivation presented here is less intuitive than the light-cone approach, it serves to illustrate that the result obtained in Sec. II is gauge invariant and independent of the light-cone framework.

While there exist several proposals to measure transversity $\delta q(x) = H_T(x,0,0)$, it is not obvious how the other chirally odd GPDs which enter our relation (38) can be directly measured in an experiment. However, it should be straightforward to determine these quantities in lattice QCD calculations [29], which would provide valuable information about the correlation between angular momentum and spin of the quarks in an unpolarized target. In addition, as we will discuss in the next section, the Boer-Mulders effect may provide valuable information about the form factor entering Eq. (48). Although this effect will not allow for a quantitative experimental determination of the relevant moments of chirally odd GPDs, it could provide useful information on the sign and rough scale of these observables. With this combined information it should be possible to add another important piece of information to our understanding of the spin structure of the nucleon.

V. BOER-MULDERS EFFECT

In analogy to the Sivers effect, where quarks in a transversely polarized target have a transverse momentum asymmetry which is perpendicular to the nucleon spin $\mathbf{S}$, it has been suggested that there could also be an asymmetry of the transverse momentum of the quarks perpendicular to the quark spin $\mathbf{s}$ in an unpolarized target [25]

$$\text{Sivers:} \qquad f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp)\cdot\mathbf{S}}{M} \tag{49}$$

$$\text{Boer-Mulders:} \qquad f_{q^\uparrow/p}(x,\mathbf{k}_\perp) = \frac{1}{2}\left[f_1^q(x,\mathbf{k}_\perp^2) - h_1^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp)\cdot\mathbf{s}}{M}\right]. \tag{50}$$

Here $f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2)$ and $h_1^{\perp q}(x, \mathbf{k}_\perp^2)$ are referred to as the Sivers and Boer-Mulders function respectively. Both the Sivers as well as the Boer-Mulders function require a nontrivial FSI. In Refs. [21] it has been suggested that the transverse distortion of impact parameter dependent (unpolarized) quark distributions in a transversely polarized target can give rise to a Sivers effect. If the quarks before they are being knocked out of the nucleon in SIDIS have a preferential direction in position space then the FSI can translate this position space asymmetry into a momentum space asymmetry. Since the FSI is expected to be attractive on average, this means that a transverse distortion in the $+\hat{x}$ direction would translate into a momentum asymmetry in the $-\hat{x}$ direction.

The distortion in impact parameter space for quarks with flavor q can be related to κ_q, i.e. the contribution to the anomalous magnetic moment (with the electric charge of the quarks factored out) from the same quark flavor [14]. Within the heuristic mechanism for the Sivers effect developed in Refs. [21, 22] one thus finds that the average Sivers effect for flavor q and κ^q should have opposite signs

$$f_{1T}^{\perp q} \sim -\kappa^q. \tag{51}$$

The signs for the predicted Sivers effect for u and d quarks in a proton have recently been confirmed by the HERMES collaboration [23]. Furthermore, the correlation above (51) has been observed in a number of toy model calculations as well [30].

As far as the transverse distortion of transverse polarized quark distributions is concerned, the forward matrix element of $2\tilde{H}_T + E_T$, i.e.

$$\kappa_T^q \equiv \int dx \left[2\tilde{H}_T(x,0,0) + E_T(x,0,0)\right] \tag{52}$$

plays a role similar to the anomalous magnetic moment κ^q for the unpolarized quark distributions on a transverse polarized target. Indeed, κ_T^q governs the transverse spin-flavor dipole moment in an unpolarized target (33). Indeed, κ_T^q tells us, in units of $\frac{1}{2M}$, how far and in which direction the average position of quarks with spin in the $\hat{x}$ direction, is shifted in the $\hat{y}$ direction for an unpolarized target relative to the transverse center of momentum.

Encouraged by the success of the impact parameter distortion based mechanism for the Sivers effect, we propose a similar semi-classical mechanism for the Boer-Mulders effect: if $\kappa_T > 0$, then the distribution for quarks polarized in the $+\hat{y}$ direction is shifted towards the $-\hat{x}$ direction (Fig. 1). The FSI is expected to have a qualitatively similar effect on deflecting this distorted position space into the opposite direction, i.e. for $\kappa_T > 0$ we expect that quarks polarized in the $+\hat{y}$ direction should be preferentially deflected in the $+\hat{x}$ direction. In accordance with the Trento convention (49) [31] this implies that $h_1^{\perp q} < 0$. More generally, we expect that on average the Boer-Mulders function for flavor q and κ_T^q should have opposite signs

$$h_1^{\perp q} \sim -\kappa_T^q. \tag{53}$$

In appendix B some of the arguments from Ref. [22] are repeated for the case of $h_1^{\perp q}$. For a more detailed discussion the reader is refered to Refs. [21, 22].

Furthermore, up to a rescaling by the factor κ_T^q/κ^q, we expect the average Boer Mulders function to be of roughly the same scale as the Sivers function.

In the case of the transverse distortion of chirally even impact parameter dependent parton distributions, the quantity that determines the magnitude of the distortion, i.e. the anomalous magnetic moment κ^q, is known experimentally (up to uncertainties from the contribution of s quarks). In the chirally odd case essentially nothing is known about the corresponding quantity $2\tilde{H}_T(x,0,0) + E_T(x,0,0)$ from experiment, although the long distance tail of chirally odd GPDs might be accessible in diffractive electroproduction of vector meson pairs [32]. Therefore it would be very useful to determine this quantity in lattcie QCD, so that at least a rough estimate can be made for sign and magnitude of the Boer-Mulders function.

VI. SUMMARY

We have studied the light-cone momentum density of a delocalized, but axially symmetric wave packet describing a transversely polarized particle that is at rest. Two effects lead to deviations from axial symmetry in the resulting momentum density. For a particle with a nontrivial internal structure (e.g. if it has an anomalous magnetic moment) there is an intrinsic asymmetry, relative to the particle's center of momentum, that is described by the GPD $E(x,0,0)$. In addition, the center of momentum of the whole wave packet is shifted sideways relative to the center of instant form wave packet by half a Compton wavelength. This sideways shift is responsible for the term proportional to $xH(x,0,0)$ in Ji's relation.

The T^{++} component of the energy momentum tensor that appears in the angular momentum relation does not mix transversity and it is therefore possible to decompose the angular momentum into transversity components. The information to be gained by performing this decomposition is the correlation between the transverse spin and the transverse angular momentum carried by the quarks. We find that the correlation between transverse spin and angular momentum of the quarks in an unpolarized target is described by a linear combination of chirally odd GPDs $H_T(x,0,0)+2\tilde{H}_T(x,0,0)+E_T(x,0,0)$.

The same linear combination of GPDs $(2\tilde{H}_T(x,0,0)+E_T(x,0,0))$ that appears in the correlation between transverse spin and angular momentum of the quarks in an unpolarized target also describes the transverse displacement of quarks with a given transversity in an unpolarized target relative to the center of momentum. We suggest that the resulting angular dependence of the chirality density, in combination with the final state interaction, gives rise to the T-odd Boer-Mulders effect and we make a prediction for the sign of the Boer-Mulders effect in terms of those GPDs. Lattice determinations of chirally odd GPDs can thus be used to predict the sign of the Boer-Mulders function. Likewise, an experimental measurement of the Boer-Mulders function could be used to learn about the correlation between transverse spin and angular momentum of the quarks in an unpolarized target.

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APPENDIX A: ANGULAR MOMENTUM AND LIGHT-CONE MOMENTUM DENSITY

The ansatz for the form factor of the energy momentum tensor (9) implicitly uses of Lorentz invariance. In this appendix, we will demonstrate how these symmetries enter the derivation of the representation of the angular momentum in terms of the impact parameter dependent light-cone momentum density T^{++}.

We start from the expectation value of $J_q^x = T_q^{0z}y - T_q^{0y}z$ taken in a delocalized wave packet at rest. We will furthermore consider the specific example where the target is an eigenstate of the total angular momentum in the $\hat{x}$-direction, i.e. up to a phase, it is invariant under rotations about the $\hat{x}$ axis. Upon performing a 90^o-rotation around the $\hat{x}$ axis one thus finds

$$\int d^3\mathbf{r} \left\langle T_q^{0y}(\mathbf{r})\right\rangle z = -\int d^3\mathbf{r} \left\langle T_q^{0z}(\mathbf{r})\right\rangle y \tag{A1}$$

and therefore

$$\left\langle J_q^x \right\rangle = 2\int d^3\mathbf{r} \left\langle T_q^{0z}(\mathbf{r})\right\rangle y. = \int d^3\mathbf{r} \left\langle \left[T_q^{0z}(\mathbf{r}) + T_q^{z0}(\mathbf{r})\right]\right\rangle y, . \tag{A2}$$

Likewise, performing a 180^o rotation around the $\hat{x}$ axis yields

$$\begin{aligned} \int d^3\mathbf{r} \left\langle T_q^{00}(\mathbf{r})\right\rangle y &= -\int d^3\mathbf{r} \left\langle T_q^{00}(\mathbf{r})\right\rangle y = 0 \\ \int d^3\mathbf{r} \left\langle T_q^{zz}(\mathbf{r})\right\rangle y &= -\int d^3\mathbf{r} \left\langle T_q^{zz}(\mathbf{r})\right\rangle y = 0 \end{aligned} \tag{A3}$$

and therefore $T_q^{0z}+T_q^{z0}$ in Eq. (A2) can be replaced by $2T_q^{++} = T_q^{0z}+T_q^{z0}+T_q^{00}+T_q^{zz}$ yielding

$$\left\langle J_q^x \right\rangle = 2\int d^3\mathbf{r} \left\langle T_q^{++}(\mathbf{r})\right\rangle y. \tag{A4}$$

Finally, for a delocalized wave packet describing a state with zero momentum, $T_q^{++}(\mathbf{r}$ is time-independent. It is thus possible to replace $\sqrt{2}\int dz \longrightarrow \int dx^-$ in these integrals, yielding a light-cone representation of the angular momentum in terms of twist-2 operators

$$\left\langle J_q^x \right\rangle = \sqrt{2}\int dx^- \int d^2\mathbf{r}_\perp \left\langle T_q^{++}(\mathbf{r})\right\rangle y \tag{A5}$$

in agreement with Eqs. (12,14).

APPENDIX B: QUARK CORRELATIONS AND THE BOER-MULDERS FUNCTION

In this appendix, we will follow the approach in Ref. [22] and relate the Boer-Mulders (BM) function to color density-density correlations in the transverse plane. The gauge invariant operator definition of the unintegrated transversity density relevant for the BM function in SIDIS reads

$$\delta^i q(x,\mathbf{k}_\perp) \equiv \int \frac{dy^- d^2\mathbf{y}_\perp}{16\pi^3} e^{-ixp^+y^- + i\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \left\langle p \left| \bar{q}_U(y) U_{[\infty^-,\mathbf{y}_\perp;\infty^-,\mathbf{0}_\perp]} \sigma^{+i}\gamma_5 q_U(0) \right| p \right\rangle , \tag{B1}$$

with

$$\begin{aligned} q_U(0) &= U_{[\infty^-,\mathbf{0}_\perp;0^-,\mathbf{0}_\perp]} q(0) & \text{(B2)}\\ \bar{q}_U(y) &= \bar{q}(y) U_{[y^-,\mathbf{y}_\perp;\infty^-,\mathbf{y}_\perp]}. & \text{(B3)} \end{aligned}$$

The U's are Wilson line gauge link, for example

$$U_{[0;\xi]} = P \exp\left(ig \int_0^1 ds \xi_\mu A^\mu(x\xi) \right) \tag{B4}$$

connecting the points 0 and ξ. The choice of paths in Eq. (B1) is not arbitrary, but reflects the final state interactions, as the ejected quark travels along the light-cone. The gauge link segment at light-cone infinity is formally necessary to render Eq. (B1) gauge invariant, but plays an important role only in the light-cone gauge $A^+ = 0$, where the segments along the light-cone do not contribute.

Following Ref. [22], we evaluate (B1) in the light-cone gauge, yielding for the average transverse momentum

$$\int d^2\mathbf{k}_\perp \delta^i q(x,\mathbf{k}_\perp)\mathbf{k}_\perp = -g \int \frac{dy^-}{4\pi} e^{-ixp^+y^-} \left\langle p \left| \bar{q}(y^-,\mathbf{0}_\perp) \frac{\lambda^a}{2} \sigma^{+i}\gamma_5 q \mathbf{A}^a_\perp(\infty^-,\mathbf{0}_\perp) \right| p \right\rangle , \tag{B5}$$

where λ^a are the Gell-Mann matrices. In Ref. [22], a constraint condition on the gauge field at $y^- = \infty$ were derived. Solving those to lowest order and inserting the result back into Eq. (B5) yields

$$\int d^2\mathbf{k}_\perp \delta^i q(x,\mathbf{k}_\perp) k^j_\perp = -\frac{g}{2} \int \frac{dy^-}{4\pi} e^{-ixp^+y^-} \int \frac{d^2\mathbf{x}_\perp}{2\pi} \langle p | \, \bar{q}(y^-,\mathbf{0}_\perp) \frac{\lambda^a}{2} \sigma^{+i}\gamma_5 q(0) \rho^a(\mathbf{x}_\perp) \, | p \rangle \, \frac{x^j}{\mathbf{x}^2_\perp} \tag{B6}$$

where

$$\rho^a(\mathbf{x}_\perp) = g \int dx^- \left[-g f^{abc} A^b_i \partial_- A^c_i + \sum_q \bar{q}\gamma^+ \frac{\lambda^a}{2} q \right] \tag{B7}$$

is the color charge density at position $\mathbf{x}_\perp$ integrated over all x^-. The average transverse momentum can thus be related to the transverse color density-density dipole-correlations between the transversity density and the spin averaged density of all partons.

Eq. (B6) is equivalent to treating the FSI in first order perturbation theory, and can, to this order, be used to justify our intuitive picture developed above. If we study the asymmetry at a relatively low Q^2 scale and/or large x, the correlation in Eq. (B6) should be dominated by the valence quarks where the Gell-Mann matrices can effectively be replaced by an overall color factor, which is negative due to the attractive nature of the QCD potential in a color singlet nucleon. Therefore, we can relate the average transverse momentum to the color neutral density-density correlation. If the chirally odd GPDs exhibit a large transverse dipole moment (κ_T large) it almost impossible not to get a significant density-density dipole-correlations between the transversity density and the spin averaged density of all partons. This is true even though we do not expect the density-density correlation to factorize. The rest of the argument is the same as in Ref. [22]. Nonperturbatively, both for $f_{1T}^\perp$ [22] as well as for $h_1^\perp$, we are guided more by intuition in order to arrive at Eqs. (51,53).

However, there is one important difference between $f_{1T}^\perp$ and $h_1^\perp$: in the case of $f_{1T}^\perp$ we were able to show that the net Sivers effect, summed over all flavors and the glue and integrated over all momenta must vanish [33]. No such statement can be made for $h_1^\perp$.

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